

Emergent gravity, violated relativity and dark matter

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Abstract

The nonlinear affine Goldstone model of the emergent gravity, built on the nonlinearly realized/hidden affine symmetry, is concisely revisited. Beyond General Relativity, the explicit violation of general invariance/relativity, under preserving general covariance, is exposed. Dependent on a nondynamical affine connection, a generally covariant second-order effective Lagrangian for metric gravity is worked out, with the general relativity violation and the gravitational dark matter serving as the signatures of emergence.

Key words: spontaneous symmetry breaking, nonlinear realizations, emergent gravity, violated relativity, dark matter

1 Introduction

It is widely accepted nowadays that General Relativity (GR) may be just (a piece of) an effective field theory of gravity to be ultimately superseded at the high energies by a more fundamental/underlying theory. At that, the conventional metric gravity could cease to be a priori existent, but, instead, would become an emergent/induced phenomenon. A lot of the drastically different approaches towards the emergence of gravity and space-time is presently conceivable.¹ In this paper, we work out an approach to the goal treating the gravity as an affine Goldstone phenomenon in the framework of the effective field theory.

As a herald of an unknown high-energy theory there typically serves at the lower energies a nonlinear model. Being based on a nonlinearly realized/hidden symmetry, remaining linear on an unbroken subgroup, such a model could encounter in a concise manner for the spontaneously/dynamically broken symmetries of the fundamental theory. Inevitably, this occurs at the cost of more uncertainty and a partial loss of content. For the global continuous internal symmetries, the nonlinear model framework was developed in [3, 4]. This approach proved to be extremely useful for studying, e.g., the so-called chiral model and played an important role in the advent of QCD as the true fundamental theory of strong interactions.

One might thus naturally expect that in the quest for an underlying theory of gravity GR should first be substituted by a nonlinear model. As such a model for gravity, aimed principally at reconstructing GR, there was originally proposed the model based on the nonlinearly realized/hidden affine symmetry, remaining linear on the unbroken

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¹For recent surveys of the emergent gravity and space-time, see, e.g. [1, 2].

Poincare subgroup [5, 6].² In the context of emergence of the gravity and space-time, the model was elaborated in [8]. At that, reproducing GR the model may well include the general invariance/relativity violation [8]–[13].³ To this end, one should envisage in a field theory two kinds of fields – the dynamical/relative and nondynamical/absolute ones – and, respectively, two kinds of the diffeomorphism symmetries. First, the kinematical symmetry – the covariance – which restricts the mathematical form of the theory. Second, the dynamical symmetry – the invariance/relativity – which serves as a gauge symmetry for gravity determining the physical content of the latter. In GR, without nondynamical fields, these notions coincide, both being the general ones. But beyond GR, in the presence of nondynamical fields, the notions differ [12, 13].⁴ For consistency, the general covariance should better be preserved. On the other hand, the GR violation may well take place, serving as a source of the gravitational dark matter (DM). In a simplest case, such a scenario was worked out for the well-defined theory of gravity minimally violating GR to the unimodular relativity, with the scalar-graviton DM [9]–[13]. Extending this scenario to other types of the GR violation and gravitational DM would thus be urgent.

In this paper, the model of emergent gravity based on the nonlinearly realized/hidden affine symmetry – the nonlinear affine Godstone model – is systematically revisited. To allow for the GR violation, two kinds of coordinates – the absolute/background and relative/observer’s ones – are envisaged. A generally covariant second-order effective Lagrangian for metric gravity, dependent on a nondynamical affine connection, is consistently worked out in the most general fashion, with a limited version discussed in more detail. The model is proposed as a prototype for the emergent gravity and space-time, with the GR violation and the gravitational DM serving as the signatures of emergence.

2 Nonlinear realizations and emergent gravity

2.1 Spontaneous symmetry breaking and nonlinear realizations

To begin with, let us shortly recapitulate the techniques of the nonlinearly realized/hidden symmetries. Let a global continuous internal symmetry G , with the dimension d_G , be spontaneously/dynamically broken, $G \rightarrow H$, to some d_H -dimensional subgroup $H \subset G$. Let $K = G/H \subset G$ be the respective d_K -dimensional (for definiteness, left) coset space consisting of the (left) coset elements $k \in K$. Then any group element $g \in G$ admits a unique (at least in a vicinity of unity) decomposition $g = kh$, with $k \in K$ and $h \in H$. Henceforth under the action of a group element g_0 , one should get $g_0 k = k' h'(g_0, k)$. The group G thus acts on k by means of the transformations $k \xrightarrow{g_0} k' = g_0 k h'^{-1}(g_0, k)$ dependent, generally, on k (henceforth the term nonlinear). Mapping a flat space R^d onto K , $R^d \rightarrow K$, defines on R^d a coset-valued field $k(\xi) \in K$, $\xi \in R^d$. This induces a nonlinear realization of G on K . Restricted by the unbroken subgroup H , i.e., under $g_0 = h_0$, the nonlinear realization of G is to be a usual linear representation of H , $k \rightarrow k' = h_0 k h_0^{-1}$, with $h'(h_0, k) = h_0$, and thus $h'(I, k) = I$. Putting $k = \exp(\sum_i \pi_i X_i)$, $i = 1, \dots, d_K$, $d_K = d_G - d_H$, with X_i being the broken generators of G , one can treat the d_K -component field π as a Goldstone boson emerging under the global symmetry breaking. Due to the isomorphism $G \simeq K \otimes H$ (at least in a vicinity of unity), one can

²For a fiber bundle formalism, cf. [7].

³The term general covariance violation used in [8]–[11] is to be more appropriately substituted by the general invariance/relativity violation [12, 13].

⁴For a discussion of the general covariance vs. general invariance, cf. also [14].

substitute a coset element k by its equivalence class κ obtained from k through the (right) multiplication by an arbitrary $h \in H$. At that, the nonlinear realization gets linearized as $\kappa \xrightarrow{g_0} \kappa' = g_0 \kappa h^{-1}$, modulo an arbitrary $h(\xi) \in H_{\text{loc}}$ independent of κ . And v.v., fixing a gauge for H_{loc} results in imposing the $d_H = d_G - d_K$ restrictions on κ and choosing thus a nonlinear realization $k \xrightarrow{g_0} k' = g_0 k h'^{-1}(g_0, k)$. Hence, being equivalent to the linear representation, all the nonlinear realizations for breaking $G \rightarrow H$ are equivalent among themselves in the effective field theory sense. At that, the linearization of a hidden symmetry may be more advantageous as embodying on par all the equivalent nonlinear realizations.

2.2 Gravity as an affine Goldstone phenomenon

In the case at hand, an underlying theory of gravity is to be originally invariant under the global affine group $G = IGL(d, R)$, $d = 4$.⁵ Eventually, the symmetry spontaneously/dynamically brakes down to the Poincare one, assumed to be exact:

$$G = IGL(d, R) \rightarrow H = ISO(1, d - 1). \quad (1)$$

A putative mechanism of such a breaking is beyond the scope of the model. According to general theory, the breaking results in the nonlinear realization of the affine symmetry on the coset space $G/H = IGL(d, R)/ISO(1, d - 1)$, with the $d(d + 1)/2$ -component coset elements. The Goldstone boson of the respective nonlinear realization is to be treated as a primary gravity field. To consistently apply the nonlinear realization technique to such a global external symmetry, a two-stage procedure is to be implemented, starting from a flat affine background and extending then to a curved one.

2.3 Flat affine background

First, let $\mathcal{R}_d \simeq R^d$ be a d -dimensional homogeneous space, with the affine group as the group of motions, $\mathcal{R}_d \rightarrow \mathcal{R}_d$. By default, \mathcal{R}_d admits the globally affine coordinates $\xi^m \in R^d$, $m = 0, 1, \dots, n - 1$,⁶ undergoing the affine transformations:

$$\xi^m \xrightarrow{(A, a)} \xi'^m = \xi^n A_n^{-1m} + a^m, \quad (2)$$

with the arbitrary constant parameters A_n^m and a^m for the (reversible) linear deformations and translations, respectively.⁷ This space will serve as the representation one for constructing the nonlinear model. In accord with the general formalism there are two modes for realization of the hidden affine symmetry: the nonlinear and linearized ones.

2.3.1 Pseudo-symmetric nonlinear realization

A coset element ϑ_m^a , $a = 0, 1, \dots, d - 1$, may uniquely be chosen to be pseudo-symmetric, i.e.,

$$\eta^{am} \vartheta_m^b = \eta^{bm} \vartheta_m^a \quad (3)$$

(at least in a suitable neighbourhood of $\vartheta_m^a = \delta_m^a$, where this condition is evidently fulfilled). Here, η^{ab} (and η_{ab}) is the invariant under $SO(1, d - 1)$ Minkowski symbol, by

⁵The following consideration is formally independent of $d \geq 2$.

⁶Being here just a notation, the index $m = 0$ is understood to subsequently compile with the unbroken Lorentz subgroup.

⁷Remaining unbroken, the translation part of the symmetry is omitted in what follows.

which the globally Lorentzian indices a, b , etc., are manipulated. Under $A \in GL(d, R)$ the coset element should transform nonlinearly as

$$\vartheta_m^a(\xi) \xrightarrow{A} \vartheta_m'^a(\xi') = A_m^n \vartheta_n^b(\xi) \Lambda_b'^{-1a}(A, \vartheta), \quad (4)$$

with $\Lambda' \in SO(1, d-1) \subset GL(d, R)$ chosen so to retain the pseudo-symmetry after action of A . At that, due to $\Lambda_{ab} = \Lambda_{ba}^{-1}$ there automatically fulfills the linearity condition $\Lambda'(\Lambda, \vartheta) = \Lambda$ for any $\Lambda \in SO(1, d-1)$.⁸ Present the symmetric Lorentz tensor $\vartheta^{ab} \equiv \eta^{am} \vartheta_m^b$ in terms of a symmetric tensor h^{ab} as $\vartheta \equiv \exp(h/2)$, where, e.g., $(hh)^{ab} = h^{ac} h^{bd} \eta_{cd}$, etc. With $H = SO(1, d-1)$ serving as a classification group, one can treat h^{ab} , with $d(d+1)/2$ independent components, as a tensor Goldstone boson emerging under the breaking of the global affine symmetry to the Poincare one.

2.3.2 Locally Lorentzian linear representation

The affine Goldstone model gets simplified with the nonlinear realization being linearized in terms of a d^2 -component frame-like field ϑ_m^α , $\alpha = 0, 1, \dots, n-1$ (and its inverse ϑ_α^m). The field transforms under $A \in GL(d, R)$ as

$$\vartheta_m^\alpha(\xi) \xrightarrow{A} \vartheta_m'^\alpha(\xi') = A_m^n \vartheta_n^\beta(\xi) \Lambda_\beta^{-1\alpha}(\xi) \quad (5)$$

(and likewise for ϑ_m^α), modulo an arbitrary $\Lambda_\alpha^\beta(\xi) \in SO(1, d-1)_{\text{loc}}$ satisfying $\Lambda_{\alpha\beta} = \Lambda_{\beta\alpha}^{-1}$, with the invariant $\eta_{\alpha\beta}$. Due to invariance under $SO(1, d-1)_{\text{loc}}$, with the $d(d-1)/2$ local parameters, the number of the independent components remains in fact the same $d(d+1)/2$. Explicitly, one can impose a Lorentz gauge by projecting $\vartheta_m^\alpha \rightarrow \vartheta_m^a = \vartheta_m^\beta \Lambda_\beta^{-1a}(\vartheta)$ so that ϑ_m^a becomes pseudo-symmetric. Thus, the two realization modes, the nonlinear and linearized ones, are equivalent. However, the linear representation may be more advantageous due to grasping all the equivalent nonlinear realizations.

Restricting ourselves to the pure gravity, introduce the generic action element produced by an infinitesimal neighbourhood $d^d\xi$ of a reference point Ξ as follows:

$$dS = \mathcal{L}_g d^d\xi \equiv L_g(\vartheta_m^\alpha, \partial_n \vartheta_m^\alpha, \dots) |\det(\vartheta_m^\alpha)| d^d\xi, \quad (6)$$

with dS being invariant relative to $GL(d, R) \otimes SO(1, n-1)_{\text{loc}}$. For the pure gravity, the field ϑ_m^α may be converted into the symmetric Lorentz-invariant second-rank affine tensor g_{mn} with $d(d+1)/2$ independent degrees of freedom:

$$g_{mn}(\xi) = \vartheta_m^\alpha \eta_{\alpha\beta} \vartheta_n^\beta. \quad (7)$$

In particular, one has $|\det(\vartheta_m^\alpha)| = \sqrt{|\det(g_{mn})|}$. This tensor will eventually be treated as the emergent metric being absent prior to the affine symmetry breaking. Transforming the result to the arbitrary curvilinear coordinates x^μ on \mathcal{R}_d and integrating over \mathcal{R}_d one would get the nonlinear model of gravity on the flat affine background. To account for a more general topology, one should go over to a curved affine background, with the action element (6) used as a flat counterpart.

⁸Moreover, the dilatations R are represented linearly, too, with the trivial compensating factor, $\Lambda'(R, \vartheta) = I$.

2.4 Curved affine background

Let now \mathcal{M}_d be a d -dimensional differentiable “world” manifold marked with some “observer’s” coordinates $x^\mu \in R^d$, $\mu = 0, 1, \dots, n-1$.⁹ Let \mathcal{M}_d be moreover endowed with a nondynamical affine connection $\hat{\Gamma}_{\mu\nu}^\lambda(x)$, free of torsion, $\hat{\Gamma}_{\mu\nu}^\lambda - \hat{\Gamma}_{\nu\mu}^\lambda = 0$. Such a background affine structure is to be formed by an underlying theory on par with the affine symmetry breaking.¹⁰ In a vicinity of a fixed, but otherwise arbitrary point X the connection may be decomposed as follows

$$\hat{\Gamma}_{\mu\nu}^\lambda(x) = \hat{\Gamma}_{\mu\nu}^\lambda(X) + \frac{1}{2} \hat{R}_{\mu\rho\nu}^\lambda(X)(x - X)^\rho + \mathcal{O}((x - X)^2), \quad (8)$$

where $\hat{R}_{\mu\rho\nu}^\lambda(X)$ is the background curvature tensor in the reference point X . In a patch around X , choose on \mathcal{M}_d some coordinates $\xi_X^m = \xi_X^m(x)$ (having the inverse $x^\mu = x^\mu(\xi_X)$) so that conventionally

$$\hat{\Gamma}_{mn}^l(\xi_X) = e_m^\mu e_n^\nu e_\lambda^l(x) \left(\hat{\Gamma}_{\mu\nu}^\lambda(x) - e_r^\lambda(x) \partial^2 \xi_X^r / \partial x^\mu \partial x^\nu \right), \quad (9)$$

where $e_\mu^m(x) = \partial \xi_X^m / \partial x^\mu$ and $e_m^\mu(x) = \partial x^\mu / \partial \xi_X^m|_{\xi_X = \xi_X(x)}$. Differentiating the reversibility relations $\xi_X^m(x(\xi_X)) = \xi_X^m$ and $x^\mu(\xi_X(x)) = x^\mu$ one gets $e_\lambda^m e_n^\lambda = \delta_n^m$ and $e_l^\mu e_\nu^l = \delta_\nu^\mu$. More particularly, adjust the coordinates ξ_X^m as follows:

$$\xi_X^l = \Xi_X^l + e_\lambda^l(X) \left((x - X)^\lambda + \frac{1}{2} \hat{\Gamma}_{\mu\nu}^\lambda(X) (x - X)^\mu (x - X)^\nu \right) + \mathcal{O}((x - X)^3), \quad (10)$$

implying

$$e_l^\lambda(X) \partial^2 \xi_X^l / \partial x^\mu \partial x^\nu|_{x=X} = \hat{\Gamma}_{\mu\nu}^\lambda(X). \quad (11)$$

In view of (9) one thus gets $\hat{\Gamma}_{mn}^l(\Xi_X) = 0$. Moreover, in view of (8) for coordinates ξ_X^m one has

$$\hat{\Gamma}_{mn}^l(\xi_X) = \frac{1}{2} \hat{R}_{mrn}^l(\Xi_X) (\xi_X - \Xi_X)^r + \mathcal{O}((\xi_X - \Xi_X)^2). \quad (12)$$

The manifold \mathcal{M}_d looking in the coordinates ξ_X^m approximately flat around Ξ_X , call such coordinates the locally affine ones in the point X . In these coordinates, project a patch of \mathcal{M}_d around X onto the representation space \mathcal{R}_d (associated with the tangent space in X) through $\xi^m - \Xi_X^m = \xi_X^m - \Xi_X^m + \mathcal{O}((\xi_X - \Xi_X)^2)$. This maps in the leading approximation the action element (6) from \mathcal{R}_d onto \mathcal{M}_d . Transform then the local result around Ξ_X from coordinates ξ_X^m to the arbitrary observer’s coordinates x^μ by means of substitutions $\vartheta_m^\alpha = e_m^\mu \vartheta_\mu^\alpha$, $\partial_m = e_m^\mu \partial_\mu$ and $d^d \xi_X = |\det(e_\mu^m)| d^d x$, with frames $e_\mu^m(x)$ and $e_m^\mu(x)$. Integrating finally over \mathcal{M}_d one gets the generic gravity action (redefining $X \rightarrow x$) as follows:

$$S = \int L_g(\vartheta_\mu^\alpha, \partial_\nu \vartheta_\mu^\alpha, \dots; \hat{\Gamma}_{\mu\nu}^\lambda) |\det(\vartheta_\mu^\alpha)| d^d x. \quad (13)$$

Evidently, the locally Lorentzian frame ϑ_μ^α satisfies the reversibility relations $\vartheta_\mu^\alpha \vartheta_\beta^\mu = \delta_\beta^\alpha$ and $\vartheta_\alpha^\mu \vartheta_\nu^\alpha = \delta_\nu^\mu$. Due to $\vartheta_\mu^\alpha = e_\mu^m \vartheta_m^\alpha$ the frame transforms under a diffeomorphism $x^\mu \rightarrow x'^\mu = x'^\mu(x)$ as

$$\vartheta_\mu^\alpha(x) \rightarrow \vartheta'_\mu{}^\alpha(x') = \frac{\partial x'^\nu}{\partial x^\mu} \vartheta_\nu^\beta(x) \Lambda_\beta^{-1\alpha}(x), \quad (14)$$

⁹The index $\mu = 0$ here is just a notation acquiring the physical meaning after the emergence of metric.

¹⁰This reflects the assumption of the absence of a prior metric structure on an underlying level. Only an affine texture of the background is supposed.

modulo an arbitrary $\Lambda \in SO(1, d-1)_{\text{loc}}$ (and likewise for $\vartheta_\alpha^\mu = e_m^\mu \vartheta_\alpha^m$). The action (13), as originated from (6), is to be invariant relative to the local Lorentz transformations, being at the same time generally covariant. At that, the Lorentzian frame ϑ_μ^α is to be treated as the dynamical one, while the background affine connection $\hat{\Gamma}_{\mu\nu}^\lambda$ as nondynamical. Otherwise, under extremizing S , the background connection should not be varied, $\delta \hat{\Gamma}_{\mu\nu}^\lambda = 0$. Under the requirement of the background-independence, the action S would preserve the general diffeomorphism invariance/relativity. In the case of the residual dependence on the background $\hat{\Gamma}_{\mu\nu}^\lambda$, the action, though retaining the general covariance, violates, partially or completely, the general invariance/relativity.

2.4.1 Emergent metric

Restricting himself by the pure gravity one can equivalently choose as an independent variable for gravity, instead of the Lorentzian frame ϑ_μ^α , its bilinear Lorentz-invariant combination

$$g_{\mu\nu}(x) = \vartheta_\mu^\alpha \eta_{\alpha\beta} \vartheta_\nu^\beta = e_\mu^m \vartheta_m^\alpha \eta_{\alpha\beta} \vartheta_n^\beta e_\nu^n = e_\mu^m g_{mn} e_\nu^n, \quad (15)$$

with $|\det(\vartheta_\mu^\alpha)| = \sqrt{|\det(g_{\mu\nu})|}$.¹¹ At that, the frame ϑ_μ^α , of which $g_{\mu\nu}$ is composed, though being the primary gravity field, manifests itself explicitly only in interactions with matter (omitted here). The tensor field $g_{\mu\nu}$ may be treated as an emergent metric. It is the emergence of metric, which converts a background manifold \mathcal{M}_d with an affine connection into the true space-time, the latter becoming in a sense emergent, as well.¹²

3 Violated relativity and dark matter

3.1 Affine symmetry

Building the proper nonlinear model starts out from \mathcal{R}_d in the globally affine coordinates ξ^m . To construct the Lagrangian dependent on the affine tensor g_{mn} and its derivatives construct first the Christoffel-like affine tensor

$$\Gamma_{mn}^l(\xi) = \frac{1}{2} g^{lk} (\partial_m g_{nk} + \partial_n g_{mk} - \partial_k g_{mn}). \quad (16)$$

A derivative of g_{mn} may uniquely be expressed through a combination of Γ_{mn}^l (and v.v.). By means of the latter, one can construct the Riemann tensor R_{mrn}^l , the Ricci tensor $R_{mn} = R_{mln}^l$ and the Ricci scalar $R = g^{mn} R_{mn}$. Altogether, one can construct on \mathcal{R}_d the generic affine-invariant second-order effective Lagrangian for gravity

$$L_g = \frac{1}{2} \kappa_g^2 \left(L_0 + \sum_{i=1}^5 \varepsilon_i \Delta L_i \right), \quad (17)$$

with the partial bi-linear in Γ_{mn}^l contributions as follows:

$$\begin{aligned} L_0 &= 2\Lambda - R(g_{mn}, \Gamma_{mn}^l), & \Delta L_1 &= g^{mn} \Gamma_{mk}^k \Gamma_{nl}^l, \\ \Delta L_2 &= g_{mn} g^{kl} g^{rs} \Gamma_{kl}^m \Gamma_{rs}^n, & \Delta L_3 &= g^{mn} \Gamma_{mn}^k \Gamma_{kl}^l, \\ \Delta L_4 &= g_{mn} g^{kl} g^{rs} \Gamma_{kr}^m \Gamma_{ls}^n, & \Delta L_5 &= g^{mn} \Gamma_{mk}^l \Gamma_{nl}^k. \end{aligned} \quad (18)$$

¹¹By this token, one can use the convectional relation $e_\mu^m = g^{mn} g_{\mu\nu} e_\nu^n$, etc.

¹²In the same vein, one could formally consider other patterns of the affine symmetry breaking, $GL(d, R) \rightarrow SO(n, d-n)$, $n = 0, 1, \dots, [d/2]$, resulting in the same number of the affine Goldstone bosons and the emergent metric with the space-time signature $(n, d-n)$, to be eventually selected [8]. For $d = 4$, cf., e.g. [15].

For completeness, there is included into L_0 a constant Λ . The parameter $\kappa_g = 1/\sqrt{8\pi G}$ is the Planck mass, with G being the Newton's constant, and ε_i , $i = 1, \dots, 5$, are the dimensionless free parameters. The presented terms exhaust all the bilinear in Γ_{mn}^l second-order ones admitted by the affine symmetry, with no prior preference among them.¹³ Transforming the results to the curvilinear coordinates x^μ on \mathcal{R}_d one would arrive at a generally covariant theory of gravity on a flat affine background. However, this is just a limited version of a more general case (see, further on).

3.2 General covariance

Let us then map the above results from \mathcal{R}_d onto \mathcal{M}_d , first, in the locally affine coordinates ξ_X^m around Ξ_X through the identical substitution $\Gamma_{mn}^l(\xi) \rightarrow \Gamma_{mn}^l(\xi_X)$ and then in the arbitrary observer's coordinates x^μ around X (using the counterpart of (9) for $\Gamma_{mn}^l(\Xi_X)$ supplemented by (11)). Altogether, we get the relation

$$\Gamma_{mn}^l(\Xi_X) = e_m^\mu e_n^\nu e_\lambda^l(X) \left(\Gamma_{\mu\nu}^\lambda(X) - \hat{\Gamma}_{\mu\nu}^\lambda(X) \right), \quad (19)$$

where conventionally

$$\Gamma_{\mu\nu}^\lambda(x) = \frac{1}{2} g^{\lambda\rho} (\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}) \quad (20)$$

(with $X \rightarrow x$) is the Christoffel connection corresponding to metric $g_{\mu\nu}(x)$. Altogether, the most general second-order generally covariant effective Lagrangian for the emergent metric gravity is given by (17), with¹⁴

$$\begin{aligned} L_0 &= 2\Lambda - R(g_{\mu\nu}, \Gamma_{\mu\nu}^\lambda), & \Delta L_1 &= g^{\mu\nu} B_{\mu\kappa}^\kappa B_{\nu\lambda}^\lambda, \\ \Delta L_2 &= g_{\mu\nu} g^{\kappa\lambda} g^{\rho\sigma} B_{\kappa\lambda}^\mu B_{\rho\sigma}^\nu, & \Delta L_3 &= g^{\mu\nu} B_{\mu\nu}^\kappa B_{\kappa\lambda}^\lambda, \\ \Delta L_4 &= g_{\mu\nu} g^{\kappa\lambda} g^{\rho\sigma} B_{\kappa\rho}^\mu B_{\lambda\sigma}^\nu, & \Delta L_5 &= g^{\mu\nu} B_{\mu\kappa}^\lambda B_{\nu\lambda}^\kappa, \end{aligned} \quad (21)$$

dependent on the generally covariant tensor¹⁵

$$B_{\mu\nu}^\lambda(x) \equiv \Gamma_{\mu\nu}^\lambda - \hat{\Gamma}_{\mu\nu}^\lambda. \quad (22)$$

The background-independent term L_0 corresponds to GR with a cosmological constant Λ , while the background-dependent ones ΔL_i , $i = 1, \dots, 5$, to the GR violation.¹⁶ The theory of gravity given by the GR-violating effective Lagrangian (21) and (22) may be referred to as Violated Relativity (VR). Most generally, it depends on the $d^2(d+1)/2$ nondynamical functions besides the five constant Lagrangian parameters ε_i .¹⁷

¹³On the affine symmetry reason, one could add two linear terms, $g^{mn} \partial_l \Gamma_{mn}^l$ and $g^{mn} \partial_m \Gamma_{nl}^l$, which however may be expressed through the rest of the terms modulo surface contributions.

¹⁴The terms without derivatives of $g_{\mu\nu}$, as well as the extra factors given by the powers of $g = \det(g_{\mu\nu})$, are forbidden by the hidden affine symmetry.

¹⁵On the covariance reason, such a dependence was postulated in [10].

¹⁶At that, under the restriction ab initio by GR, the flat affine background would superficially suffice.

¹⁷Treating ΔL_i as the small perturbations and L_0 as the leading term, one can extend the latter by the higher-order generally covariant contributions.

3.3 Gravitational DM

Varying the gravity action with respect to $g_{\mu\nu}$, under fixed $\hat{\Gamma}_{\mu\nu}^\lambda$, we get the vacuum gravity field equations in a generic form as follows:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{\kappa_g^2} \sum_i^5 \varepsilon_i \Delta T_{\mu\nu}^{(i)}(g_{\rho\sigma}, B_{\rho\sigma}^\lambda) \equiv \frac{1}{\kappa_g^2} \Delta T_{\mu\nu}. \quad (23)$$

Here $\Delta T_{\mu\nu}^{(i)}$ are the generally covariant contributions to the equations due to ΔL_i :

$$\Delta T^{(i)\mu\nu} = \frac{2}{\sqrt{|g|}} \frac{\delta(\sqrt{|g|}\Delta L_i)}{\delta g_{\mu\nu}}, \quad (24)$$

with $g = \det(g_{\mu\nu})$ and $\delta/\delta g_{\mu\nu}$ designating the total variational derivative. The r.h.s. of (23) may formally be treated as the covariantly conserved, $\nabla_\mu \Delta T^{\mu\nu} = 0$, canonical energy-momentum tensor of the gravitational DM due to the GR violation ($\varepsilon_i \neq 0$). The affine “texture” of space-time with $\hat{\Gamma}_{\mu\nu}^\lambda$, mimicking such DM, signifies ultimately the gravity and space-time as emergent.

3.4 Limited GR violation

Generally, the phenomenological study of VR is rather cumbersome. To simplify it as much as possible, consider the formal limit $\hat{\Gamma}_{\mu\nu}^\lambda = 0$ corresponding to the flat affine background in the globally affine coordinates. The various observations being in reality fulfilled in the different coordinates, it is practically impossible for an observer to guess/use the unknown globally affine coordinates ab initio. In the lack of this knowledge, one should start from suitable observer’s coordinates x^μ , assuming some $\hat{\Gamma}_{\mu\nu}^\lambda(x)$, to eventually reveal, with the help of observations, the globally affine coordinates ξ^m (independent of any reference point X), so that $\hat{\Gamma}_{mn}^l(\xi) \equiv 0$. According to (9), one should have in this case the restriction

$$\hat{\Gamma}_{\mu\nu}^\lambda(x) = \frac{\partial^2 \xi^l}{\partial x^\mu \partial x^\nu} \frac{\partial x^\lambda}{\partial \xi^l} \Big|_{\xi=\xi(x)} = \hat{e}_l^\lambda \partial_\mu \hat{e}_\nu^l. \quad (25)$$

Here $\hat{e}_\mu^m = \partial \xi^m / \partial x^\mu$ and $\hat{e}_l^\lambda = \partial x^\lambda / \partial \xi^l|_{\xi=\xi(x)}$ are the frames relating the distinguished globally affine coordinates and the arbitrary observer’s ones in terms of the d nondynamical generally covariant scalar fields $\xi^m(x)$ (having the inverse $x^\mu(\xi)$). This is the least set of free functions to consistently account for the GR violation, under preserving general covariance. The topology of the affine background, flat vs. curved, is thus not a pure theoretical question but becomes, in principle, liable to observational verification. Anyhow, the flat affine background determined by (25) could be treated as an approximation, in a space-time region, to a curved affine background determined by a regular $\hat{\Gamma}_{\mu\nu}^\lambda$.¹⁸

3.5 Unimodular relativity

For ΔL_1 one has

$$B_{\mu\lambda}^\lambda = \partial_\mu \ln \sqrt{|g|} - \hat{\gamma}_\mu, \quad (26)$$

¹⁸A limiting generally noncovariant case corresponding here, in this paper, to GR violation with $\hat{\Gamma}_{\mu\nu}^\lambda = 0$, $\xi^m = \delta_\mu^m x^\mu$ and $\hat{e}_\mu^m = \delta_\mu^m$ was elaborated, irrespective of DM, in [16].

where $\hat{\gamma}_\mu \equiv \hat{\Gamma}_{\mu\lambda}^\lambda$. Moreover, if the affine background is flat, one gets in view of (25):

$$\hat{\gamma}_\mu = \hat{e}_i^\lambda \partial_\mu \hat{e}_\lambda^i = \partial_\mu \ln |\det(\hat{e}_\lambda^i)|. \quad (27)$$

Thus under this limitation one has

$$\Delta L_1 = g^{\mu\nu} \partial_\mu \varsigma \partial_\nu \varsigma, \quad (28)$$

where

$$\varsigma = \ln(\sqrt{|g|}/\hat{\mu}), \quad (29)$$

with

$$\hat{\mu} = |\det(\partial_\mu \xi^m)|. \quad (30)$$

The field ς , determined by the ratio of the two scalar densities of the same weight, behaves like a generally covariant scalar. The nondynamical field $\hat{\mu}$ being a unimodular scalar, the theory with only ΔL_1 (in addition to L_0) may for uniformity be referred to as Unimodular Relativity (UR), with the scalar graviton ς serving as the gravitational DM [9]–[13]. Now, the scalar graviton is nothing but a Goldstone boson corresponding to the hidden dilatation symmetry.¹⁹ Besides, the so-called “modulus” $\hat{\mu}$ acquires the clear-cut physical meaning.^{20, 21} To qualitatively compile with the astrophysical data on the galaxy anomalous rotation curves due to the dark halos, there should fulfill $\varepsilon_1^{1/2} \sim v_\infty \sim 10^{-3}$, with v_∞ being an asymptotic rotation velocity. To tame phenomenologically the possible unwanted properties of the gravitational DM, a hierarchy of the GR violations $|\varepsilon_{2-5}| \ll |\varepsilon_1| \ll 1$ may be envisaged, with GR reduced first to UR and ultimately to VR.

4 Conclusion

The nonlinear affine Goldstone model may provide a prototype for the emergent gravity, with the GR violation and the gravitational DM serving as the signatures of emergence. Resulting in VR, given by the Lagrangian (21) and (22) (supplemented in the limited version by the relation (25)), the model widely extends the phenomenological horizons beyond GR, with the possible reduction of GR first to UR and then to VR. In an ultimate theoretical perspective, the model may, hopefully, serve as a guide towards a putative underlying theory of gravity and space-time.

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¹⁹Such a specific dilaton in disguise, representing a compression gravity mode in metric, may be called the “systolon” [12].

²⁰In this particular case, it proves that the unknown modulus $\hat{\mu}$ may be hidden into ς taken as an independent variable and, after finding the latter together with metric through the field equations, be reconstructed as a consistency condition. The proper solutions may be associated with DM [9]–[13]. The generally covariant formalism is crucial to this point.

²¹A theory of gravity (the so-called “TDiff gravity”) in the generally noncovariant form corresponding here, in this paper, to UR in the gauge $\hat{\mu} = 1$ was elaborated, irrespective of DM, in [17].

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